Cylinders

Cons. of T-energy

$$\rho C\left(\frac{\partial T}{\partial t} + \underline{V}\cdot\nabla T\right) = k\nabla^2 T + q'''$$

- No $q''' = 0$
- Steady state $\partial T/\partial t = 0$
- Solid $\underline{V} = 0$
- $k, \rho, C_p = \text{¢}$

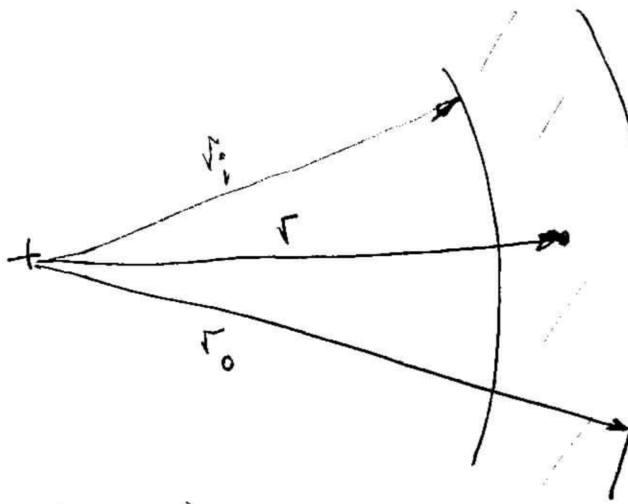
Left with $\nabla^2 T = 0$ or $\frac{1}{r}\frac{\partial}{\partial r}\left(k r \frac{\partial T}{\partial r}\right) = 0$

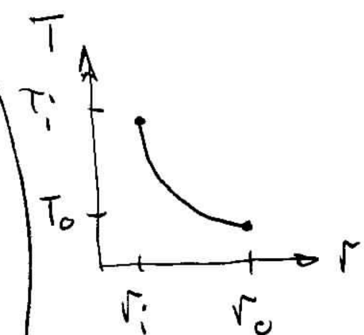
$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) = 0$$

B.C.

$$T\Big|_{r=r_i} = T_i$$

$$T\Big|_{r=r_o} = T_o$$





Solution is $T = \dfrac{(T_i - T_o)}{\ln(r_i/r_o)}\ln\left(r/r_o\right) + T_o$

or $\boxed{\dfrac{T - T_o}{T_i - T_o} = \dfrac{\ln(r/r_o)}{\ln(r_i/r_o)}}$ Temp Dist

So then heat flux is

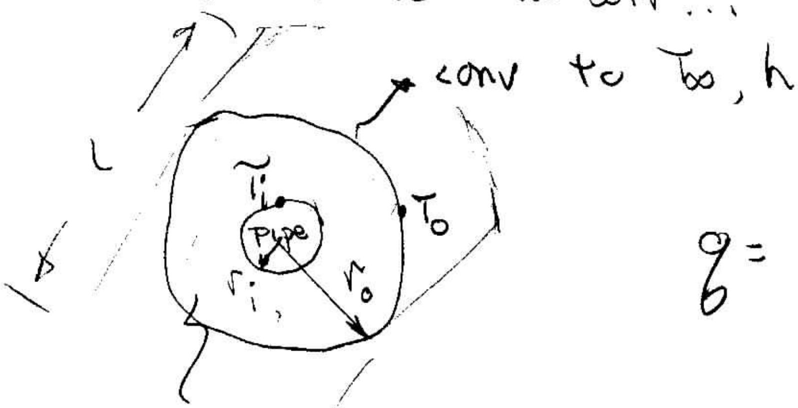
$$q\Big|_{r=r_o} = -kA\frac{\partial T}{\partial r}\Big|_{r=r_o} = -k(2\pi r L)\frac{\partial T}{\partial r}\Big|_{r=r_o} = \frac{2\pi k L\,(T_i - T_o)}{\ln(r_o/r_i)}$$

Resistance for Cyl. (per unit length)

$$q' = \frac{\Delta T}{R'} = \frac{2\pi k L \,\Delta T}{\ln(r_o/r_i)} \left(\frac{1}{L}\right) \Rightarrow \boxed{R' = \frac{\ln(r_o/r_i)}{2\pi k}}$$

or $\boxed{R = \frac{\ln(r_o/r_i)}{2\pi k L}}$

Interesting problem... insulation around a pipe with convection to the air...



insulation around the pipe

$$q = \frac{\Delta T}{R_{insul} + R_{conv}}$$

$$\frac{(T_i - T_\infty)}{\dfrac{\ln(r_o/r_i)}{2\pi k L} + \dfrac{1}{h(2\pi r_o L)}}$$

This makes $q(r_o)$... min/max prob from Calc I.

Calc $\frac{dq}{dr_o} = 0$ solve for $\boxed{r_{o,crit} = \frac{k}{h}}$

- if | $r_i < r_{crit}$
 $q\uparrow$ with more insul. | $r_i > r_{crit}$
 $q\downarrow$ with more insul. |

- Requires exterior surface to ↑ with additional insul.!

This can also be done in spherical...

$$\nabla^2 T = 0 \qquad \boxed{\frac{1}{r^2}\frac{\partial}{\partial r}\left(k r^2 \frac{\partial T}{\partial r}\right) = 0}$$

Similar B.C. $T\big|_{r=r_i} = T_i$ and $T\big|_{r=r_o} = T_o$

Temp. solution is $$T = T_o - (T_i - T_o)\frac{\left(1 - r_i/r\right)}{\left(1 - r_i/r_o\right)}$$

$$q = \frac{4\pi k (T_i - T_o)}{\left(\frac{1}{r_i} - \frac{1}{r_o}\right)}$$

$$R = \frac{\left(\frac{1}{r_i}\right) - \left(\frac{1}{r_o}\right)}{4\pi k}$$

$$r_{crit} = 2k/h$$

$\boxed{\text{HW}}$ work out details for spherical